Dr. Amarendra Kumar,
Deptt. of Chemistry,
Jain College, Ara.

1. <u>Angular Momentum operators</u> ($\hat{L}$) or $\hat{L}_x, \hat{L}_y, \hat{L}_z$

Classically, angular momentum is given by the vector product of position ($\vec{r}$) and linear momentum ($\vec{P}$) as

$$\vec{L} = \vec{r} \times \vec{P}$$

If $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along x, y and z co-ordinates, we have

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z \quad \text{and}$$

$$\vec{P} = \vec{i}P_x + \vec{j}P_y + \vec{k}P_z$$

Thus,

$$\vec{L} = (\vec{i}x + \vec{j}y + \vec{k}z) \times (\vec{i}P_x + \vec{j}P_y + \vec{k}P_z)$$

or, $$\vec{L} = \vec{i}(yP_z - zP_y) + \vec{j}(zP_x - xP_z) + \vec{k}(xP_y - yP_x) \quad \text{---} (1)$$

Also, by definition

$$\vec{L} = \vec{i}L_x + \vec{j}L_y + \vec{k}L_z \quad \text{---} (2)$$

from eqn (1) and (2), we get

$$\hat{L}_x = y\hat{P}_z - z\hat{P}_y = \frac{h}{2\pi i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$\hat{L}_y = z\hat{P}_x - x\hat{P}_z = \frac{h}{2\pi i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$\hat{L}_z = x\hat{P}_y - y\hat{P}_x = \frac{h}{2\pi i}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

2. Linear Momentum operator ($\hat{P}_x$)

For an electron wave, the wave function is represented by

$$\psi(x) = A e^{\pm \frac{2\pi i x}{\lambda}}$$

Differentiating the above eqn. with respect to x, we get

$$\frac{\partial \psi(x)}{\partial x} = \pm \frac{2\pi i}{\lambda}\left(A e^{\pm \frac{2\pi i x}{\lambda}}\right) = \pm \frac{2\pi i}{\lambda}\psi(x)$$

From de-Broglie equation, we have

$$\lambda = \frac{h}{P_x}$$

Thus,

$$\frac{d\psi(x)}{\partial x} = \pm \frac{2\pi i}{h} P_x \psi(x)$$

or, $$P_x \psi(x) = \pm \frac{h}{2\pi i} \frac{\partial \psi(x)}{\partial x}$$

Removing $\psi(x)$ from both the sides, we get the linear momentum operator as

$$\hat{P}_x = \pm \frac{h}{2\pi i} \frac{\partial}{\partial x}$$

The operator in + direction is $\frac{h}{2\pi i} \frac{\partial}{\partial x}$

The operator in − direction is $-\frac{h}{2\pi i} \frac{\partial}{\partial x}$

—x—